Dynamics of rotation of blades of wind turbines

Dr. Andrei Pavlov

Angular momentum of rotating blades of a wind turbine is **L** = I **ω**, where **ω** is the angular speed and I is the moment of inertia. The moment of inertia is specific for each system and does not change. The mechanism of generation of energy of a wind blade system is described by the classic formula

$\frac{dL}{dt} = \tau$, where τ is the torque acting on the system of blades.

With zero load (no connection to a turbine) there are two opposite components of the torque. The first component τ_1 (positive) is obtained from incoming air flow interacting with the blades and the second component τ_2 (negative) is obtained from air resistance during rotation of the blades. When the rotation is slow the first component dominates. When rotation accelerates the second component increases until the two components compensate each other $\tau_1 = \tau_2$. Then the blades rotate steadily with constant angular velocity ω and the condition $\frac{dL}{dt} = 0$ is satisfied. This is known as the law of conservation of angular momentum. The angular momentum is a function of the angular speed ω and moment of inertia. As the moment of inertia is constant, the angular momentum can be controlled by the angular speed only. Suppose that at zero load the angular momentum of the blade system is L_0 and the angular speed is ω_0, $\tau_{10} = \tau_{20}$. When the blades are connected to a turbine, a part of the angular momentum transforms to the turbine. This reduces the angular momentum of the blades. The the change of the angular momentum is described by the equation

$\frac{dL}{dt} = \tau_1 - \tau_2 - \tau_t$. When the system reaches the equilibrium at steady rotation $\tau_1 - \tau_2 - \tau_t = 0$, the angular momentum will have a value $L_1 < L_0$. At these conditions the blades can generate electrical power characterised by the torque $\tau_t = \tau_1 - \tau_2$. However, there is a threshold value of the angular momentum L_{th} below which the system will stop its movement. The threshold value is caused by forces reducing energy of the system. These are friction in mechanisms, energy losses, mechanical deformation of the parts etc. The threshold value is characterised with a minimum rotational speed ω_{th}. The angular momentum of the solid body is written by the equation

$$L = \int_V dV \rho(r,\alpha)(\vec{r} \times \vec{v})$$

The torque produced by the incoming air flow τ_1 is

$$\tau_1 = \int_S d\vec{F} \times \vec{r},$$

where *S* is the surface of the blade. The torque is obtained by integration over the entire working surface.

Our integrated blade comprises two coupled blades, and the working surface of each of component blades is about two times wider than a traditional blade. So, the torque of the integrated blade at first approximation is $\tau_{1ib} = k_1 * k_2 * \tau_{1e} \approx 4\,\tau_{1e}$, where the torque τ_{1e} is produced on a traditional blade of the same length, $k_1=2$ is the increase of width and $k_2=2$ is the number of blades in the single frame. In practice, the equations are more complicated due to the complex shape of the blades, change of the angle between the air flow and the surface of the blade during rotation. However, a general picture will remain the same.