

2.2.2025 KH

Tällainen tehtävä DeepSeekille. Vedin hatusta nuo neljä pistettä

kehitä yhtälö x-y koordinaatistoon, joka kulkee pisteiden (2,3), (4,4), (7,6) ja (9,5) kautta jollakin tarkkuudella

Thought for 270 seconds

Okay, I need to figure out an equation that passes through the points (2,3), (4,4), (7,6), and (9,5) with some accuracy. Let me start by plotting these points in my mind to see if they form a pattern. Let's see:

- At $x=2, y=3$

- $x=4, y=4$

- $x=7, y=6$

- $x=9, y=5$

Hmm, plotting these might show a trend that goes up and then down. Maybe a quadratic equation? Because quadratics can have a maximum or minimum point. Let me check if these points could lie on a parabola.

First, let's assume the equation is quadratic: $y = ax^2 + bx + c$. We have four points, but a quadratic is determined by three coefficients, so it might not pass through all four exactly. But since the problem says "with some accuracy," maybe a best-fit quadratic?

Alternatively, maybe a cubic equation? A cubic polynomial would have four coefficients ($ax^3 + bx^2 + cx + d$), so it could pass through all four points exactly. Let me try that.

Set up the system of equations using each point:

For (2,3):

$$3 = a(2)^3 + b(2)^2 + c(2) + d \rightarrow 8a + 4b + 2c + d = 3$$

For (4,4):

$$4 = a(4)^3 + b(4)^2 + c(4) + d \rightarrow 64a + 16b + 4c + d = 4$$

For (7,6):

$$6 = a(7)^3 + b(7)^2 + c(7) + d \rightarrow 343a + 49b + 7c + d = 6$$

For (9,5):

$$5 = a(9)^3 + b(9)^2 + c(9) + d \rightarrow 729a + 81b + 9c + d = 5$$

Now, we have four equations:

1. $8a + 4b + 2c + d = 3$

2. $64a + 16b + 4c + d = 4$

3. $343a + 49b + 7c + d = 6$

4. $729a + 81b + 9c + d = 5$

This system can be solved using linear algebra methods. Let's subtract equation 1 from equation 2 to eliminate d:

Equation 2 - Equation 1:

$$(64a - 8a) + (16b - 4b) + (4c - 2c) + (d - d) = 4 - 3$$

$$56a + 12b + 2c = 1 \rightarrow \text{Let's call this equation 5.}$$

Similarly, subtract equation 2 from equation 3:

Equation 3 - Equation 2:

$$(343a - 64a) + (49b - 16b) + (7c - 4c) + (d - d) = 6 - 4$$

$$279a + 33b + 3c = 2 \rightarrow \text{Equation 6.}$$

Subtract equation 3 from equation 4:

Equation 4 - Equation 3:

$$(729a - 343a) + (81b - 49b) + (9c - 7c) + (d - d) = 5 - 6$$

$$386a + 32b + 2c = -1 \rightarrow \text{Equation 7.}$$

Now we have three equations (5, 6, 7):

5. $56a + 12b + 2c = 1$

6. $279a + 33b + 3c = 2$

7. $386a + 32b + 2c = -1$

Let's try to eliminate c. Multiply equation 5 by 3 and equation 6 by 2 to make coefficients of c equal:

$$\text{Equation 5} * 3: 168a + 36b + 6c = 3 \rightarrow \text{Equation 8}$$

$$\text{Equation 6} * 2: 558a + 66b + 6c = 4 \rightarrow \text{Equation 9}$$

Subtract equation 8 from equation 9:

$$(558a - 168a) + (66b - 36b) + (6c - 6c) = 4 - 3$$

$$390a + 30b = 1 \rightarrow \text{Simplify by dividing by 30:}$$

$$13a + b = 1/30 \rightarrow \text{Equation 10}$$

Now, let's use equations 5 and 7 to eliminate c. Subtract equation 5 from equation 7:

$$(386a - 56a) + (32b - 12b) + (2c - 2c) = -1 - 1$$

$$330a + 20b = -2 \rightarrow \text{Divide by 10:}$$

$$33a + 2b = -0.2 \rightarrow \text{Equation 11}$$

Now we have equations 10 and 11:

$$10. 13a + b = 1/30 \approx 0.0333\dots$$

$$11. 33a + 2b = -0.2$$

Let's solve this system. From equation 10, express $b = 1/30 - 13a$

Substitute into equation 11:

$$33a + 2*(1/30 - 13a) = -0.2$$

$$33a + 2/30 - 26a = -0.2$$

$$(33a - 26a) + 1/15 = -0.2$$

$$7a + 1/15 = -0.2$$

Convert 0.2 to 1/5, so:

$$7a = -1/5 - 1/15 = (-3/15 - 1/15) = -4/15$$

$$a = (-4/15)/7 = -4/(15*7) = -4/105 \approx -0.0381$$

Now, substitute a back into equation 10:

$$13*(-4/105) + b = 1/30$$

$$-52/105 + b = 1/30$$

Convert to common denominator, which is 210:

$$-52*2/210 + b = 7/210$$

$$-104/210 + b = 7/210$$

$$b = (7 + 104)/210 = 111/210 = 37/70 \approx 0.5286$$

Now, go back to equation 5 to find c:

$$56a + 12b + 2c = 1$$

$$56*(-4/105) + 12*(37/70) + 2c = 1$$

Calculate each term:

$$56*(-4)/105 = -224/105 = -32/15 \approx -2.1333$$

$$12*(37/70) = 444/70 = 222/35 \approx 6.3429$$

So:

$$-32/15 + 222/35 + 2c = 1$$

Convert to common denominator 105:

$$(-32*7)/105 + (222*3)/105 + 2c = 1$$

$$-224/105 + 666/105 + 2c = 1$$

$$(666 - 224)/105 + 2c = 1$$

$$442/105 + 2c = 1$$

$$442/105 \approx 4.2095$$

So:

$$2c = 1 - 4.2095 \approx -3.2095$$

$$c \approx -1.6048$$

But let's do exact fractions:

$$442/105 = 442 \div 105 = 4 + 22/105$$

$$1 = 105/105$$

So:

$$2c = 105/105 - 442/105 = -337/105$$

$$c = -337/(105*2) = -337/210 \approx -1.6048$$

Now, go back to equation 1 to find d:

$$8a + 4b + 2c + d = 3$$

Substitute $a = -4/105$, $b = 37/70$, $c = -337/210$

Calculate each term:

$$8a = 8*(-4/105) = -32/105 \approx -0.3048$$

$$4b = 4*(37/70) = 148/70 = 74/35 \approx 2.1143$$

$$2c = 2*(-337/210) = -674/210 = -337/105 \approx -3.2095$$

So:

$$-32/105 + 74/35 - 337/105 + d = 3$$

Convert all to 105 denominator:

$$-32/105 + (74 \cdot 3)/105 - 337/105 + d = 3$$

$$-32 + 222 - 337 \text{ all over } 105 + d = 3$$

$$(-32 + 222) = 190; 190 - 337 = -147$$

So:

$$-147/105 + d = 3$$

$$\text{Simplify } -147/105 = -49/35 = -7/5 \approx -1.4$$

So:

$$-7/5 + d = 3 \rightarrow d = 3 + 7/5 = 22/5 = 4.4$$

Therefore, the cubic equation is:

$$y = (-4/105)x^3 + (37/70)x^2 + (-337/210)x + 22/5$$

Let me check if this passes through all four points.

Test $x=2$:

$$y = (-4/105)(8) + (37/70)(4) + (-337/210)(2) + 22/5$$

Calculate each term:

$$-32/105 \approx -0.3048$$

$$(37 \cdot 4)/70 = 148/70 \approx 2.1143$$

$$(-337 \cdot 2)/210 = -674/210 \approx -3.2095$$

$$22/5 = 4.4$$

$$\text{Sum: } -0.3048 + 2.1143 - 3.2095 + 4.4 \approx (-0.3048 - 3.2095) + (2.1143 + 4.4) \approx -3.5143 + 6.5143 \approx 3 \rightarrow$$

Correct.

Test $x=4$:

$$y = (-4/105)(64) + (37/70)(16) + (-337/210)(4) + 22/5$$

$$-256/105 \approx -2.4381$$

$$(37 \cdot 16)/70 = 592/70 \approx 8.4571$$

$$(-337 \cdot 4)/210 \approx -1348/210 \approx -6.4190$$

$$22/5 = 4.4$$

$$\text{Sum: } -2.4381 + 8.4571 - 6.4190 + 4.4 \approx (8.4571 - 2.4381) + (-6.4190 + 4.4) \approx 6.019 - 2.019 \approx 4 \rightarrow$$

Correct.

$x=7$:

$$y = (-4/105)(343) + (37/70)(49) + (-337/210)(7) + 22/5$$

$$-1372/105 \approx -13.0667$$

$$(37 \cdot 49)/70 = 1813/70 \approx 25.9$$

$$(-337 \cdot 7)/210 \approx -2359/210 \approx -11.2333$$

$$22/5 = 4.4$$

$$\text{Sum: } -13.0667 + 25.9 - 11.2333 + 4.4 \approx (-13.0667 - 11.2333) + (25.9 + 4.4) \approx -24.3 + 30.3 \approx 6 \rightarrow$$

Correct.

$x=9$:

$$y = (-4/105)(729) + (37/70)(81) + (-337/210)(9) + 22/5$$

$$-2916/105 \approx -27.7714$$

$$(37 \cdot 81)/70 = 2997/70 \approx 42.8143$$

$$(-337 \cdot 9)/210 \approx -3033/210 \approx -14.4429$$

$$22/5 = 4.4$$

$$\text{Sum: } -27.7714 + 42.8143 - 14.4429 + 4.4 \approx (-27.7714 - 14.4429) + (42.8143 + 4.4) \approx -42.2143 + 47.2143 \approx 5 \rightarrow \text{Correct.}$$

Wow, the cubic passes through all four points exactly. So the equation is:

$$y = (-4/105)x^3 + (37/70)x^2 - (337/210)x + 22/5$$

Alternatively, converting to decimal for easier reading:

$$a \approx -0.0381, b \approx 0.5286, c \approx -1.6048, d \approx 4.4$$

$$\text{So } y \approx -0.0381x^3 + 0.5286x^2 - 1.6048x + 4.4$$

But since the question is in Finnish and mentions "jollakin tarkkuudella" (with some accuracy), maybe decimal approximation is acceptable. Alternatively, leave it in fractions.

Alternatively, if a cubic is too complex, maybe a quadratic best fit. Let's see:

If we try a quadratic fit, even though there are four points, which would require a cubic for exactness.

Compute the quadratic regression.

Formulae for quadratic regression:

Given n points, the coefficients a , b , c of $y = ax^2 + bx + c$ satisfy:

$$\Sigma y = a\Sigma x^2 + b\Sigma x + cn$$

$$\Sigma xy = a\Sigma x^3 + b\Sigma x^2 + c\Sigma x$$

$$\Sigma x^2 y = a\Sigma x^4 + b\Sigma x^3 + c\Sigma x^2$$

Compute the sums:

First, list the points:

(2,3), (4,4), (7,6), (9,5)

$$n=4$$

Compute Σx , Σy , Σx^2 , Σx^3 , Σx^4 , Σxy , $\Sigma x^2 y$.

$$\Sigma x = 2+4+7+9=22$$

$$\Sigma y = 3+4+6+5=18$$

$$\Sigma x^2 = 4 + 16 + 49 + 81 = 150$$

$$\Sigma x^3 = 8 + 64 + 343 + 729 = 1144$$

$$\Sigma x^4 = 16 + 256 + 2401 + 6561 = 9234$$

$$\Sigma xy = (2*3)+(4*4)+(7*6)+(9*5) = 6 + 16 + 42 + 45 = 109$$

$$\Sigma x^2 y = (4*3)+(16*4)+(49*6)+(81*5) = 12 + 64 + 294 + 405 = 775$$

Now, set up the equations:

$$1. 18 = a*150 + b*22 + c*4$$

$$2. 109 = a*1144 + b*150 + c*22$$

$$3. 775 = a*9234 + b*1144 + c*150$$

Now, we have the system:

$$150a + 22b + 4c = 18 \rightarrow \text{Equation A}$$

$$1144a + 150b + 22c = 109 \rightarrow \text{Equation B}$$

$$9234a + 1144b + 150c = 775 \rightarrow \text{Equation C}$$

Let's solve this system. Maybe using elimination.

First, equation A multiplied by something to eliminate c.

Let's try to eliminate c first. Multiply equation A by 5.5 to get 22c:

But perhaps better to use matrix methods or substitution.

Alternatively, write in matrix form:

Coefficient matrix:

$$[150 \ 22 \ 4]$$

$$[1144 \ 150 \ 22]$$

$$[9234 \ 1144 \ 150]$$

Constants: $[18, 109, 775]$

Alternatively, use Cramer's rule, but that might be tedious.

Alternatively, subtract equations to eliminate variables.

From Equation A: $150a + 22b + 4c = 18$

Equation B: $1144a + 150b + 22c = 109$

Equation C: $9234a + 1144b + 150c = 775$

Let's try to eliminate c. Multiply Equation A by 5.5 (11/2) to get 22c:

Multiply Equation A by 11/2:

$$(150 \cdot 11/2)a + (22 \cdot 11/2)b + (4 \cdot 11/2)c = 18 \cdot 11/2$$

Which is:

$$825a + 121b + 22c = 99$$

Now subtract this from Equation B:

Equation B - (825a + 121b + 22c = 99):

$$(1144a - 825a) + (150b - 121b) + (22c - 22c) = 109 - 99$$

$$319a + 29b = 10 \rightarrow \text{Equation D}$$

Similarly, eliminate c between Equation B and C.

Multiply Equation B by (150/22) to get 150c:

But messy. Alternatively, multiply Equation B by 75/11 to make 150c:

Alternatively, let's do step by step.

From Equation B: $1144a + 150b + 22c = 109$

Multiply Equation B by (150/22) to get 150c:

But maybe better to take Equation B and multiply by something, and Equation C to eliminate c.

Alternatively, use Equation A and Equation B to express c in terms of a and b.

From Equation A:

$$4c = 18 - 150a - 22b \rightarrow c = (18 - 150a - 22b)/4$$

Substitute this into Equation B:

$$1144a + 150b + 22 * ((18 - 150a - 22b)/4) = 109$$

Simplify:

$$1144a + 150b + (22/4)(18 - 150a - 22b) = 109$$

$$22/4 = 11/2$$

So:

$$1144a + 150b + (11/2)*18 - (11/2)*150a - (11/2)*22b = 109$$

Calculate each term:

$$(11/2)*18 = 99$$

$$(11/2)*150a = 825a$$

$$(11/2)*22b = 121b$$

Thus:

$$1144a + 150b + 99 - 825a - 121b = 109$$

Combine like terms:

$$(1144a - 825a) + (150b - 121b) + 99 = 109$$

$$319a + 29b + 99 = 109$$

$$319a + 29b = 10 \rightarrow \text{Which is the same as Equation D.}$$

$$\text{Now, Equation D: } 319a + 29b = 10$$

Similarly, eliminate c from Equation B and C.

$$\text{From Equation B: } 1144a + 150b + 22c = 109$$

$$\text{From Equation C: } 9234a + 1144b + 150c = 775$$

Multiply Equation B by (150/22) to match the c coefficient:

$$\text{Equation B} * (150/22):$$

$$1144*(150/22)a + 150*(150/22)b + 22*(150/22)c = 109*(150/22)$$

Simplify:

$$(1144*150)/22 a + (150*150)/22 b + 150c = (109*150)/22$$

Calculate coefficients:

$$1144/22=52, \text{ so } 52*150=7800a$$

$$150/22=75/11, \text{ so } 75/11*150=11250/11 b$$

$$109*150=16350, 16350/22\approx 743.1818$$

So Equation becomes:

$$7800a + (11250/11)b + 150c = 16350/22$$

Now subtract this from Equation C:

$$9234a + 1144b + 150c - [7800a + (11250/11)b + 150c] = 775 - 16350/22$$

Simplify:

$$(9234a - 7800a) + [1144b - (11250/11)b] + 0 = 775 - 743.1818$$

Calculate each term:

$$9234 - 7800 = 1434a$$

$$1144b - (11250/11)b = (1144*11 - 11250)/11 b = (12584 - 11250)/11 b = 1334/11 b$$

$$775 - 16350/22 \approx 775 - 743.1818 \approx 31.8182$$

So:

$$1434a + (1334/11)b = 31.8182$$

Multiply both sides by 11 to eliminate denominator:

$$1434*11a + 1334b = 31.8182*11$$

$$15774a + 1334b \approx 350$$

Now, we have:

$$\text{Equation E: } 15774a + 1334b \approx 350$$

$$\text{And Equation D: } 319a + 29b = 10$$

Now, solve this system. Let's write Equation D and E:

$$\text{D: } 319a + 29b = 10$$

$$\text{E: } 15774a + 1334b = 350$$

Let's solve for a and b.

First, multiply Equation D by 1334/29 to align coefficients:

$$29b = 10 - 319a$$

Multiply by (1334/29):

$$29b \cdot (1334/29) = 1334b = (10 - 319a) \cdot (1334/29)$$

$$\text{So } 1334b = (10 \cdot 1334)/29 - (319 \cdot 1334/29)a$$

Calculate:

$$10 \cdot 1334 = 13340; 13340/29 \approx 460$$

$$319 \cdot 1334 = 319 \cdot 1334 = \text{let's compute:}$$

$$319 \cdot 1000 = 319,000$$

$$319 \cdot 300 = 95,700$$

$$319 \cdot 34 = 10,846$$

$$\text{Total} = 319,000 + 95,700 = 414,700 + 10,846 = 425,546$$

$$\text{Then divide by 29: } 425,546/29 \approx 14,674$$

So:

$$1334b \approx 460 - 14,674a$$

Now plug into Equation E:

$$15774a + (460 - 14,674a) = 350$$

$$15774a - 14,674a + 460 = 350$$

$$(15774 - 14674)a = 350 - 460$$

$$1100a = -110 \rightarrow a = -110/1100 = -0.1$$

Now, substitute $a = -0.1$ into Equation D:

$$319 \cdot (-0.1) + 29b = 10$$

$$-31.9 + 29b = 10 \rightarrow 29b = 41.9 \rightarrow b = 41.9/29 \approx 1.4448 \approx 1.445$$

Now, find c from Equation A:

$$150a + 22b + 4c = 18$$

$$150 \cdot (-0.1) + 22 \cdot (1.445) + 4c = 18$$

$$-15 + 31.79 + 4c \approx 18$$

$$16.79 + 4c \approx 18 \rightarrow 4c \approx 1.21 \rightarrow c \approx 0.3025$$

Thus, the quadratic equation is:

$$y = -0.1x^2 + 1.445x + 0.3025$$

Check if this fits the points approximately.

At $x=2$:

$$y = -0.1 \cdot (4) + 1.445 \cdot 2 + 0.3025 = -0.4 + 2.89 + 0.3025 \approx 2.7925 \approx 2.8 \text{ (actual 3)}$$

$x=4$:

$$y = -0.1 \cdot 16 + 1.445 \cdot 4 + 0.3025 = -1.6 + 5.78 + 0.3025 \approx 4.4825 \approx 4.5 \text{ (actual 4)}$$

$x=7$:

$$y = -0.1 \cdot 49 + 1.445 \cdot 7 + 0.3025 = -4.9 + 10.115 + 0.3025 \approx 5.5175 \approx 5.5 \text{ (actual 6)}$$

$x=9$:

$$y = -0.1 \cdot 81 + 1.445 \cdot 9 + 0.3025 = -8.1 + 13.005 + 0.3025 \approx 5.2075 \approx 5.2 \text{ (actual 5)}$$

So the quadratic approximation has some error but is close. The cubic passes exactly, but is more complex.

The problem states "jollakin tarkkuudella" which could mean either an exact equation (like the cubic) that passes through all points, or a simpler approximation like the quadratic. However, since the cubic does pass through all four points exactly, and the user provided four points, which is exactly the number needed to determine a cubic, the correct answer is the cubic equation.

But let me confirm once more that the cubic coefficients are correct. Plugging in $x=9$:

$$y = (-4/105)(729) + (37/70)(81) + (-337/210)(9) + 22/5$$

Calculate each term:

$$-4/105 \cdot 729 = -2916/105 = -972/35 \approx -27.7714$$

$$37/70 \cdot 81 = 2997/70 \approx 42.8143$$

$$-337/210 \cdot 9 = -3033/210 = -1011/70 \approx -14.4429$$

$$22/5 = 4.4$$

$$\text{Sum: } -27.7714 + 42.8143 - 14.4429 + 4.4 \approx (-27.7714 - 14.4429) + (42.8143 + 4.4) = -42.2143 + 47.2143 = 5 \rightarrow \text{Correct.}$$

So the cubic is exact. Therefore, the equation is the cubic polynomial found earlier.

In fractional form:

$$y = (-4/105)x^3 + (37/70)x^2 - (337/210)x + 22/5$$

Alternatively, simplifying fractions:

$-4/105$ can be reduced? 4 and 105 have GCD 1, so no.

37/70 is reduced.

337 and 210: GCD of 337 and 210. 337 divided by 210 is 1 with remainder 127. Then 210 and 127: 127 into 210 is 1 rem 83. 127 and 83: 83 into 127 rem 44. 83 and 44: 44 into 83 rem 39. 44 and 39: 39 into 44 rem 5. 39 and 5: 5 into 39 rem 4. 5 and 4: GCD 1. So 337/210 is reduced.

22/5 is already simple.

Alternatively, write all terms with denominator 210:

$$-4/105 = -8/210$$

$$37/70 = 111/210$$

$$-337/210 \text{ stays}$$

$$22/5 = 924/210$$

So:

$$y = (-8/210)x^3 + (111/210)x^2 - (337/210)x + 924/210$$

Combine terms:

$$y = [-8x^3 + 111x^2 - 337x + 924] / 210$$

So the equation can be written as:

$$y = \frac{-8x^3 + 111x^2 - 337x + 924}{210}$$

Alternatively, factor numerator if possible. Let me check if the numerator can be factored.

Check if $x=2$ is a root of the numerator:

$$\begin{aligned} -8*(8) + 111*(4) - 337*(2) + 924 &= -64 + 444 - 674 + 924 = (-64 - 674) + (444 + 924) = -738 \\ &+ 1368 = 630 \neq 0 \end{aligned}$$

Not a root. Maybe not factorable easily. So leave as is.

Therefore, the exact equation is the cubic polynomial above.

Since the problem allows for "jollakin tarkkuudella", which might imply an approximate equation, but since an exact one exists, it's better to provide that.

The cubic polynomial that passes exactly through the points (2,3), (4,4), (7,6), and (9,5) is:

$$y = \frac{-8x^3 + 111x^2 - 337x + 924}{210}$$

Step-by-Step Explanation:

1. Determine the Polynomial Degree:

Since there are four points, a cubic polynomial (degree 3) can pass through all points exactly.

2. Set Up the System of Equations:

Substitute each point into the general cubic equation $y = ax^3 + bx^2 + cx + d$:

$$\begin{cases} 8a + 4b + 2c + d = 3 \\ 64a + 16b + 4c + d = 4 \\ 343a + 49b + 7c + d = 6 \\ 729a + 81b + 9c + d = 5 \end{cases}$$

3. Solve the System Using Elimination:

Subtract consecutive equations to eliminate d and reduce to three equations. Continue eliminating variables step-by-step to find:

$$a = -\frac{4}{105}, \quad b = \frac{37}{70}, \quad c = -\frac{337}{210}, \quad d = \frac{22}{5}$$

4. Construct the Equation:

Substitute the coefficients back into the cubic polynomial:

$$y = -\frac{4}{105}x^3 + \frac{37}{70}x^2 - \frac{337}{210}x + \frac{22}{5}$$

5. Verify with Given Points:

Plugging each x -value into the equation confirms that the corresponding y -values match exactly.

Final Answer:

$$y = -\frac{4}{105}x^3 + \frac{37}{70}x^2 - \frac{337}{210}x + \frac{22}{5}$$